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JEE MAIN 2018
Test Paper Code – D
Questions with Solutions

MATHEMATICS

1. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is

(1) $\frac{9}{2}$ (2) 6 (3) $\frac{7}{2}$ (4) 4

Ans (1)

$y^2 = 6x \qquad 2y\frac{dy}{dx} = 6 \quad m_1 = \frac{6}{2y}$

$9x^2 + by^2 = 16$

$18x + 2by\frac{dy}{dx} = 0$

$y\frac{dy}{dx} = \frac{-18x}{2b}$

$m_2 = \frac{-18x}{2by}$

$m_1m_2 = -1 \quad \frac{-18x}{2by}.\frac{6}{2y} = -1$

$\frac{18x \times 6}{4by^2} = +1$

$\frac{18x \times 6}{4b \times 6x} = 1$

$4b = 18$

$b = \frac{18}{4} = \frac{9}{2}$

2. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ = $\vec{b}=\hat{j}+\hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u},\vec{b}=24$, then $|\vec{u}|^2$ is equal to:

(1) 84 (2) 336 (3) 315 (4) 256

Ans (2)

$\vec{u}=p\vec{a}+q\vec{b}$

$\vec{u}.\vec{a}=0$

$p|a|^2+qb.a=0$

$p(4+9+1)+q(3-1)=0$

$14p+2q=0 \qquad 7p+q=0$

$\vec{u}.\vec{b}=24$

$24=p(\vec{a}.\vec{b})+a|\vec{b}|^2$

$=2p+2q$

$p+q=12$

$7p+q=0$

$p=-2\ \ q=14$

$\vec{u}=p\vec{a}+q\vec{b}=-4i-6j+2k+14j+14k$

$=-4i+8j+16k$

$|\vec{u}|^2=16+64+256=336$

3. For each $t\in R$, let [t] be the greatest integer less than or equal to t. Then $\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+....+\left[\frac{15}{x}\right]\right)$

(1) does not exist (in R). (2) is equal to 0.

(3) is equal to 15. (4) is equal to 120.

Ans (4)

Consider $\frac{1}{x}\le\left[\frac{1}{x}\right]<\frac{1}{x}+1$

$\Rightarrow \frac{r}{x}\le\left[\frac{r}{x}\right]<\frac{r}{x}+1 \qquad r=1,2,......15$

$\Rightarrow \sum_{r=1}^{15}\frac{r}{x}\le\sum_{r=1}^{15}\left[\frac{r}{x}\right]<\sum_{r=1}^{15}\left(\frac{r}{x}+1\right)$

$\Rightarrow \sum_{r=1}^{15}\frac{r}{x}\le\sum_{r=1}^{15}\left[\frac{r}{x}\right]<\sum_{r=1}^{15}\frac{r}{x}+120$

$\Rightarrow \sum_{r=1}^{15}x\frac{r}{x}\le\sum_{r=1}^{15}x\left[\frac{r}{x}\right]<\sum_{r=1}^{15}x\frac{r}{x}+120x$

Taking $\lim_{x\to 0^+} r=1$

$\Rightarrow \lim_{x\to 0^+}\sum_{r=1}^{15}r\le\lim_{x\to 0^+}\sum_{r=1}^{15}x\left[\frac{r}{x}\right]<\lim_{x\to 0^+}\left(\sum_{r=1}^{5}r+120x\right)$

$\Rightarrow \frac{15(16)}{2}\le\lim_{x\to 0^+}\sum_{r=1}^{15}x\left[\frac{r}{x}\right]<\lim_{x\to 0^+}\frac{15(16)}{2}$

$\therefore \lim_{x\to 0^+}\sum_{r=1}^{15}x\left[\frac{r}{x}\right]=120$

4. If L_1 is the line of intersection of the planes $2x-2y+3z-2=0$, $x-y+z+1=0$ and L_2 is the line of intersection of the planes $x+2y-z-3=0$, $3x-y+2z-1=0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is:

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{4\sqrt{2}}$ (3) $\frac{1}{3\sqrt{2}}$ (4) $\frac{1}{2\sqrt{2}}$

Ans (3)

L_1 is intersection of $2x - 2y + 3z - 2 = 0$ and $x - y + z + 1 = 0$

$$\therefore \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i} + \hat{j}$$

$\therefore$ Direction ratios of L_1 are 1, 1, 0

To find a point on L_1, put $x = 0$

$\therefore -2y + 3z = 2$

and $-y + z = -1$

Solving these, we get $(x, y, z) \equiv (0, 5, 4)$

Similarly, L_2 is intersection of $x + 2y - z - 3 = 0$ and $3x - y + 2z - 1 = 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 3\hat{i} - 5\hat{j} - 7\hat{k}$$

$\therefore$ Direction ratios of L_2 are 3, –5, –7

$\therefore$ Equation of a plane containing L_1 and L_2 is

$$\begin{vmatrix} x-0 & y-5 & z-4 \\ 1 & 1 & 0 \\ 3 & -5 & -7 \end{vmatrix} = 0$$

$\therefore -7x + 7y - 8z - 3 = 0$

$\therefore$ Distance from origin $= \frac{|-3|}{\sqrt{7^2+7^2+8^2}}$

$= \frac{1}{3\sqrt{2}}$

5. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{8}$ (3) $\frac{\pi}{2}$ (4) 4π

Ans (1)

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx \quad \ldots (1)$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^{-x}} dx \quad \left(\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx \right)$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2^x \sin^2 x}{1+2^x} dx \qquad \ldots (2)$$

Adding (1) and (2)

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x\left(1+2^x\right)}{1+2^x} dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx$$

$$2I = 2\int_{0}^{\frac{\pi}{2}} \sin^2 x dx \qquad (\because \sin^2 x \text{ is even function})$$

$$\therefore\ I = \int_{0}^{\frac{\pi}{2}} \sin^2 x dx$$

$$= \int_{0}^{\frac{\pi}{2}} \left(\frac{1-\cos 2x}{2}\right) dx$$

$$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - 0\right]$$

$$= \frac{\pi}{4}$$

6. Let $g(x) = \cos x^2, f(x) = \sqrt{x}$, and $\alpha, \beta\ (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. Units) bounded by the curve $y = (g \circ f)x$ and the lines $x = \alpha, x = \beta$ and y = 0, is

(1) $\frac{1}{2}(\sqrt{2}-1)$ (2) $\frac{1}{2}(\sqrt{3}-1)$ (3) $\frac{1}{2}(\sqrt{3}+1)$ (4) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

Ans (2)

$g(x) = \cos x^2 \qquad f(x) = \sqrt{x}$

$g[f(x)] = g\left(\sqrt{x}\right)$

$= \cos\left(\sqrt{x}\right)^2$

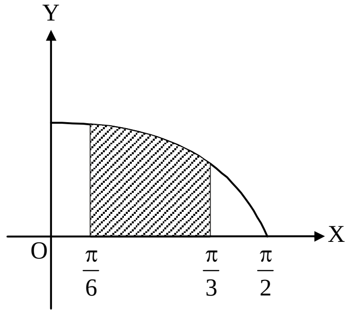
$= \cos x$

$18x^2 - 9\pi x + \pi^2 = 0$

$x = \dfrac{9\pi \pm \sqrt{81\pi^2 - 72\pi^2}}{36}$

$= \dfrac{\pi}{3}$ or $\dfrac{\pi}{6}$

Area under the curve of y = cos x bounded by $x = \frac{\pi}{6}, \frac{\pi}{3}$ and $y = 0$



$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x\, dx = \sin x]_{\pi/6}^{\pi/3}$$

$$= \frac{\sqrt{3}-1}{2}$$

7. If sum of all the solutions of the equation $8\cos x.\left(\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in [0, π] is kπ, then k is equal to

(1) $\frac{20}{9}$ (2) $\frac{2}{3}$ (3) $\frac{13}{9}$ (4) $\frac{8}{9}$

Ans (3)

$8\cos\left[\cos\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right]=1$

$8\cos\left[\left(\cos^2\frac{\pi}{6}-\sin^2 x\right)-\frac{1}{2}\right]=1$

$8\cos x\left[\frac{3}{4}-\sin^2 x-\frac{1}{2}\right]=1$

$8\cos x\left[\frac{1}{4}-\sin^2 x\right]=1$

$8\cos x\frac{[1-4\sin^2 x]}{4}=1$

$2\cos x[1-4(1-\cos^2 x)]=1$

$2\cos x[-3+4\cos^2 x]=1$

$2[4\cos^3 x-3\cos x]=1$

$2\cos 3x=1$

$\cos 3x=\frac{1}{2}$; $x\in[0,\pi]\Rightarrow 3x\in[0,3\pi]$

(i) $3x=\frac{\pi}{3}$

$x=\frac{\pi}{9}$

(ii) $3x=2\pi+\frac{\pi}{3}$

$x=\frac{2\pi}{3}+\frac{\pi}{9}$

(iii) $3x=2\pi-\frac{\pi}{3}$

$x=\frac{2\pi}{3}-\frac{\pi}{9}$

The sum of the solutions is $=\frac{\pi}{9}+\frac{2\pi}{3}+\frac{\pi}{9}+\frac{2\pi}{3}-\frac{\pi}{9}=k\pi$

$=\frac{\pi}{9}+\frac{5\pi}{3}=\frac{13\pi}{9}=k\pi\Rightarrow \boxed{k=\frac{13}{9}}$

8. Let $f(x)=x^2+\frac{1}{x^2}$ and $g(x)=x-\frac{1}{x}$ $x\in R-\{-1,0,1\}$. If $h(x)=\frac{f(x)}{g(x)}$, then the local minimum value of h (x) is

(1) $2\sqrt{2}$ (2) 3 (3) –3 (4) $-2\sqrt{2}$

Ans (1)

$$h(x)=\frac{f(x)}{g(x)}=\frac{x^2+\frac{1}{x^2}}{x-\frac{1}{x}}=\frac{\left(x-\frac{1}{x}\right)^2+2}{x-\frac{1}{x}}$$

Let $x-\frac{1}{x}=y$

$$h(x)=\frac{y^2+2}{y}=y+\frac{2}{y}$$

$AM\ge GM$

$$\frac{y+\frac{2}{y}}{2}\ge\sqrt{y\times\frac{2}{y}}$$

$$y+\frac{2}{y}\ge 2\sqrt{2}$$

9. The integral $\int\frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$ is equal to

(1) $\frac{-1}{1+\cot^3 x}+C$ (2) $\frac{1}{3(1+\tan^3 x)}+C$ (3) $\frac{-1}{3(1+\tan^3 x)}+C$ (4) $\frac{1}{1+\cot^3 x}+C$

(where C is a constant of integration)

Ans (3)

$$\int\frac{\sin^2 x\cos^2 x dx}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}$$

$$\int\frac{\sin^2 x\cos^2 x dx}{[\sin^2 x(\sin^3 x+\cos^3 x)+\cos^2 x(\sin^3 x+\cos^3 x)]^2}$$

$$=\int\frac{\sin^2 x\cos^2 x dx}{(\sin^3 x+\cos^3 x)^2}$$

$$=\int\frac{\sin^2 x\cos^2 x dx}{\cos^6 x(\tan^3 x+1)^2}$$

$$=\int\frac{\sin^2 x\ dx}{\cos^4 x\ (1+\tan^3 x)^2}$$

$$=\int\frac{\tan^2 x.\sec^2 x\ dx}{(1+\tan^3 x)^2}$$

$t=1+\tan^3 x$

$$\frac{dt}{dx}=3\tan^2 x\sec^2 x\ dx$$

$$=\frac{1}{3}\int\frac{dt}{t^2}=\frac{1}{3}\int t^{-2}dt$$

$$=-\frac{1}{3}\times t^{-1}$$

$$=-\frac{1}{3(1+\tan^3 x)}+C$$

10. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is:

(1) $\frac{3}{4}$ (2) $\frac{3}{10}$ (3) $\frac{2}{5}$ (4) $\frac{1}{5}$

Ans (3)

E_1 = Event the drawn ball is red.

E_2 = Event the drawn ball is black.

A = After returning the balls to the bag event of drawing a red ball.

$$P(E_1) = \frac{4}{10} = \frac{2}{5}$$

$$P(E_2) = \frac{6}{10} = \frac{3}{5}$$

$$P(A/E_1) = \frac{6}{12} = \frac{1}{2}$$

$$P(A/E_2) = \frac{4}{12} = \frac{1}{3}$$

$$P(A) = P(E_1).P(A/E_1) + P(E_2).P(A/E_2)$$

$$= \frac{2}{5}.\frac{1}{2} + \frac{3}{5}.\frac{1}{3}$$

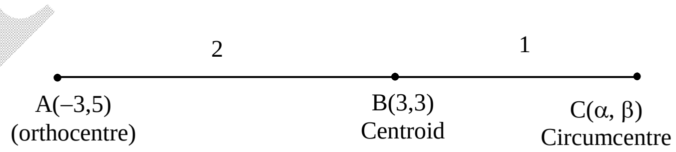
$$= \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$$

11. Let the orthocentre and centroid of a triangle he A(–3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is

(1) $\frac{3\sqrt{5}}{2}$ (2) $\sqrt{10}$ (3) $2\sqrt{10}$ (4) $3\sqrt{\frac{5}{2}}$

Ans (4)

In a triangle the orthocentre, centroid and circumcentre lie in one straight line and the centroid divides the orthocentre and circumcentre in the ratio 2 : 1.



We have $\frac{2\alpha - 3}{3} = 3, \ \frac{2\beta + 5}{3} = 3$

$\Rightarrow \alpha = 6, \beta = 2$

So $AC = \sqrt{(6+3)^2 + (2-5)^2} = \sqrt{90}$

If AC is the diameter then radius is $\frac{\sqrt{90}}{2} = 3\sqrt{\frac{5}{2}}$

12. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is

(1) 95 (2) 195 (3) 185 (4) 85

Ans (1)

Given the curve $x^2 = y - 6$

$$2x = \frac{dy}{dx}$$

So $\left.\frac{dy}{dx}\right|_{(1,7)} = 2$

The equation of the tangent is

$y-7=2(x-1)$

$2x-y+5=0 \qquad ...(1)$

This line (1) is touching the circle $x^2+y^2+16x+12y+c=0$ whose centre is $(-8,-6)$ and radius $=\sqrt{64+36-c}=\sqrt{100-c}$

So, by condition of tangency

$$\frac{|2x-8-(-6)+5|}{\sqrt{5}}=\sqrt{100-c}$$

$$\frac{|-5|}{\sqrt{5}}=\sqrt{100-c}$$

$\Rightarrow c=95$

13. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2-x+1=0$, then $\alpha^{101}+\beta^{107}$ is equal to

(1) 2 (2) -1 (3) 0 (4) 1

Ans (4)

$x^2-x+1=0$

$$x=\frac{1\pm\sqrt{3}i}{2}=-\omega,-\omega^2$$

Then $\alpha=-\omega, \beta=-\omega^2$

$$\alpha^{101}+\beta^{107}=(-\omega)^{101}+(-\omega^2)^{107}$$

$$=-\omega^{101}-\omega^{214}$$

$$=-(\omega^3)^{33}\cdot\omega^2-(\omega^3)^{-71}\cdot\omega$$

$$=-\omega^2-\omega$$

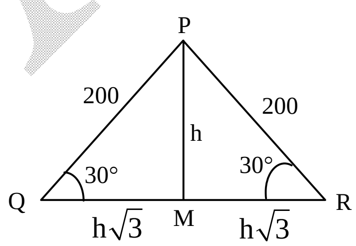
$$=1$$

14. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P. Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is

(1) $50\sqrt{2}$ (2) 100 (3) 50 (4) $100\sqrt{3}$

Ans (2)

From ΔPQM, $\tan 30°=\frac{h}{MQ}$



$$\frac{1}{\sqrt{3}}=\frac{h}{MQ}\Rightarrow MQ=\sqrt{3}h$$

Also, $h^2+3h^2=(200)^2$

$4h^2=(200)^2$

$(2h)^2=(200)^2$

$2h=200$

$h=100$ metres

15. If $\sum_{i=1}^{9}(x_i-5)=9$ and $\sum_{i=1}^{9}(x_i-5)^2=45$, then the standard deviation of the 9 items $x_1, x_2, ...x_9$ is:

(1) 3 (2) 9 (3) 4 (4) 2

Ans (4)

Let $d_i=x_i-5$

$$\therefore\ \sigma_x^2 = \sigma_d^2 = \frac{1}{9}\Sigma d_i^2 - \left(\frac{1}{9}\Sigma d_i\right)^2$$

$$= \frac{1}{9}\times 45 - \left(\frac{9}{9}\right)^2$$

$$= 5 - 1 = 4$$

$$\sigma_x = \sqrt{4} = 2$$

16. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5, (x>1)$ is

(1) 2 (2) –1 (3) 0 (4) 1

Ans (1)

$$^5C_0x^5 + {}^5C_1x^4\sqrt{x^3-1} + {}^5C_2x^3(\sqrt{x^3-1}) + {}^5C_2x^3(\sqrt{x^3-1})^3 + {}^5C_4x(\sqrt{x^3-1})^4$$

$$+ {}^5C_5\sqrt{x^3-1} + {}^5C_0x^5 - {}^5C_1x^4\sqrt{x^3-1} + {}^5C_2x^3(\sqrt{x^3-1})^2 - {}^5C_3x^2(\sqrt{x^3-1})^3$$

$$+ {}^5C_4x(\sqrt{x^3-1})^4 - {}^5C_5\sqrt{x^3-1}$$

$$= 2.{}^5C_0x^5 + 2.{}^5C_2x^3(\sqrt{x^3-1})^2 + 2.{}^5C_4x(\sqrt{x^3+1})^4$$

$$= 2[x^5 + 10x^3(x^3-1) + 5x(x^3-1)^2]$$

$$= 2[x^5 + 10x^6 - 10x^3 + 5x(x^6 - 2x^3 + 1)]$$

$$= 2[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x]$$

$$= 2x^5 + 20x^6 - 20x^3 + 10x^7 - 20x^4 + 10x$$

Sum of odd coefficient = $2 - 20 + 10 + 10 = 2$

17. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is:

(1) $36\sqrt{5}$ (2) $45\sqrt{5}$ (3) $54\sqrt{3}$ (4) $60\sqrt{3}$

Ans (2)

Hyperbola is $\frac{x^2}{9} - \frac{y^2}{36} = 1$

Equation of any tangent is $y = mx + \sqrt{9m^2 - 36}$

If this passes through T(0, 3)

$9m^2 - 36 = 9$

$\therefore$ m = $\pm\sqrt{5}$

$\therefore$ The equations of the two tangents are

$\sqrt{5}x - y + 3 = 0$ … (1) and

$\sqrt{5}x + y - 3 = 0$ … (2)

If (x_1, y_1) is the point of contact, then

Equation of tangent is $\frac{xx_1}{9} - \frac{yy_1}{36} - 1 = 0$ … (3)

Comparing (1) and (3), we get

$\frac{x_1}{9\sqrt{5}} = \frac{y_1}{36} = -\frac{1}{3}$ $\therefore$ $(x_1, y_1) = \left(-3\sqrt{5}, -12\right)$

Comparing (2) and (3), we get

$\frac{x_1}{9\sqrt{5}} = \frac{y_1}{-36} = \frac{1}{3}$ $\therefore$ $(x_1, y_1) = \left(3\sqrt{5}, -12\right)$

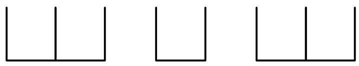
Now T ≡ (0, 3), P ≡ $\left(-3\sqrt{5}, -12\right)$ and Q ≡ $\left(3\sqrt{5}, -12\right)$

∴ Area of ΔTPQ = $\left|\frac{1}{2}\left\{(-3\sqrt{5})\,(-15) + 3\sqrt{5}\,(15)\right\}\right|$ = $45\sqrt{5}$ sq. units

18. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is

(1) at least 750 but less than 1000
(2) at least 1000
(3) less than 500
(4) at least 500 but less than 750

Ans (2)



1 dictionary can be selected from 3 in ${}^{3}C_{1}$ ways.

4 novels can be selected from 6 in ${}^{6}C_{4}$ ways

∴ Total number of such arrangement is

$4!\,({}^{6}C_{4}) \times {}^{3}C_{1} = 1080$

19. If the system of linear equations

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to

(1) 30
(2) –10
(3) 10
(4) –30

Ans (3)

AX = 0

Homogeneous equation

It has a non-zero solution $\Rightarrow |A| = 0$

$$A = \begin{bmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$$

$\Rightarrow 1(-3k + 8) - k(-9 + 4) + 3(12 - 2k) = 0$

$\Rightarrow -3k + 8 + 9k - 4k + 36 - 6k = 0$

$\Rightarrow 44 - 4k = 0$

$\Rightarrow 4k = 44$

$\Rightarrow k = 11$

$$A = \begin{bmatrix} 1 & 11 & 3 \\ 3 & 11 & -2 \\ 2 & 4 & -3 \end{bmatrix} \Rightarrow \begin{matrix} x + 11y + 3z = 0 \\ 3x + 11y - 2z = 0 \\ 2x + 4y - 3z = 0 \end{matrix}$$

Put $z = t$, $x + 11y + 3t = 0$

$3x + 11y - 2t = 0$

$$\frac{x}{-22-33}=\frac{y}{-2-9}=\frac{t}{11-33}$$

$$\frac{x}{-55}=\frac{y}{-11}=\frac{t}{-22}\Rightarrow\frac{x}{-55}=\frac{t}{-22}\Rightarrow x=\frac{-55}{-22}t=\frac{5}{2}t$$

$$\Rightarrow\frac{y}{-11}=\frac{t}{-22}\Rightarrow y=\frac{-11}{-22}t=\frac{1}{2}t$$

$$\frac{xz}{y^2}=\frac{\frac{5}{2}t.t}{\left(\frac{1}{2}t\right)^2}$$

$$=\frac{\frac{5}{2}t^2}{\frac{1}{4}t^2}$$

$$=\frac{5}{\frac{1}{2}}=10$$

20. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}=(A+Bx)(x-A)^2$, then the ordered pair (A, B) is equal to

(1) (4, 5) (2) (–4, –5) (3) (–4, 3) (4) (–4, 5)

Ans (4)

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow\begin{vmatrix} 5x-4 & 5x-4 & 5x-4 \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$$

$$\Rightarrow (5x-4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$$

$$\Rightarrow (5x-4)\begin{vmatrix} 1 & 0 & 0 \\ 2x & -(x+4) & 0 \\ 2x & 0 & -(x+4) \end{vmatrix}=(5x-4)\left((x+4)^2-0\right)$$

$= (5x-4)(x+4)^2 = (A+Bx)(x-A)^2$

$\Rightarrow A = -4,\ B = 5$

$\therefore (A, B) = (-4, 5)$

21. Two sets A and B are as under: $A=\{(a,b)\in R\times R:|a-5|<1 \text{ and } |b-5|<1\}$;
$B=\{(a,b)\in R\times R: 4(a-6)^2+9(b-5)^2\le 36\}$. Then:

(1) neither $A\subset B$ nor $B\subset A$ (2) $B\subset A$

(3) $A\subset B$ (4) $A\cap B=\phi$ (an empty set)

Ans (3)

$A = \{(a,b) \in R \times R : |a-5| < 1 \text{ and } |b-5| < 1\}$

$|a - 5| < 1 \Rightarrow a \in (4, 6)$

$|b - 5| < 1 \Rightarrow b \in (4, 6)$

$\therefore$ All the ordered pair (a, b) of set A lies inside the square whose vertices are (4, 4), (6, 4), (4, 6) and (6, 6)

$B = \{(a,b) \in R \times R : 4(a-6)^2 + 9(b-5)^2 \leq 36\}$.

$4(a - 6)^2 + 9(b - 5)^2 \leq 36$

$\Rightarrow \dfrac{(a-6)^2}{9} + \dfrac{(b-5)^2}{4} \leq 1$

This is an ellipse with centre (6, 5) and length of the major axis is 3 and minor axis is 2.

Since all the vertices of square satisfying the inequation of ellipse and are lying inside the ellipse

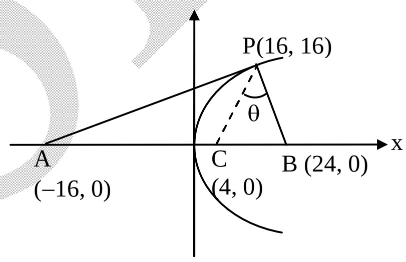
$\therefore$ A is a proper subset of B

$\therefore A \subset B$

22. Tangent and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is:

(1) $\dfrac{4}{3}$ (2) $\dfrac{1}{2}$ (3) 2 (4) 3

Ans (3)



Equation of tangent at P(16, 16) is y (16) = 8 (x + 16)

i.e., x – 2y + 16 = 0

This meets the x-axis at A(–16, 0)

Equation of normal at P is y – 16 = –2(x – 16) is 2x + y – 48 = 0

This meets the x-axis at B(24, 0)

Since $\angle APB = 90°$, mid pint of AB i.e., (4, 0) is the centre of circle through P, A and B

Slope of PC = $\dfrac{4}{3}$, slope of PB = –2

$$\therefore \tan\theta = \left|\frac{\frac{4}{3} + 2}{1 - \frac{8}{3}}\right| = 2$$

23. Let $S = \{t \in R : f(x) = |x - \pi| . (e^{|x|} - 1)\sin|x| \text{ is not differentiable at } t\}$. Then the set S is equal to

(1) $\{0, \pi\}$ (2) ϕ (an empty set) (3) $\{0\}$ (4) $\{\pi\}$

Ans (2)

$S = \{t \in R : f(x) = |x - \pi| . (e^{|x|} - 1)\sin|x| \text{ is not differentiable at } t\}$

$$f(x) = \begin{cases} (\pi - x)(e^{-x} - 1)(\sin(-x)) & \text{when } x < 0 \\ (\pi - x)(e^{x} - 1)(\sin x) & \text{when } 0 \leq x < \pi \\ (x - \pi)(e^{-x} - 1)(\sin x) & \text{when } x \geq \pi \end{cases}$$

$$= \begin{cases} (x - \pi)(e^{-x} - 1)\sin x & \text{when } x < 0 \\ (\pi - x)(e^{x} - 1)\sin x & \text{when } 0 \leq x < \pi \\ (x - \pi)(e^{x} - 1)\sin x & \text{when } x \geq \pi \end{cases}$$

$$f'(x)=\begin{cases}(e^{-x}-1)\sin x+(x-\pi)(-e^{-x})\sin x+(x-\pi)(e^{-x}-1)\cos x & \text{when } x<0\\ (-1)(e^{x}-1)\sin x+(\pi-x)e^{x}\sin x+(\pi-x)(e^{x}-1)\cos x & \text{when } 0\le x<\pi\\ (e^{x}-1)\sin x+(x-\pi)e^{x}\sin x+(x-\pi)(e^{x}-1)\cos x & \text{when } x\ge\pi\end{cases}$$

$f'(0^-)=0;\ \ f'(0^+)=0;\ \ f'(\pi^-)=0;\ \ f'(\pi^+)=0$

$\therefore$ f(x) is differentiable at all points.

24. The Boolean expression $\sim(p\vee q)\vee(\sim p\wedge q)$ is equivalent to

(1) ~ q (2) ~ p (3) p (4) q

Ans (2)

p	**q**	**p ∨ q**	**~(p∨q)**	**~p**	**~p∧q**	**~(p∨q) ∨ (~p ∧ q)**
T	T	T	F	F	F	F
T	F	T	F	F	F	F
F	T	T	F	T	T	T
F	F	F	T	T	F	T

$\therefore\ \sim(p\vee q)\vee(\sim p\wedge q)\ \equiv \sim p$

25. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is:

(1) $3x+2y=6xy$ (2) $3x+2y=6$ (3) $2x+3y=xy$ (4) $3x+2y=xy$

Ans (4)

Let slope of PQ be m

Equation of PQ is $y-y_1=m(x-x_1)$

$y-3=m(x-2)$

$mx-y=2m-3$

$$\frac{x}{\frac{2m-3}{m}}+\frac{y}{-(2m-3)}=1$$

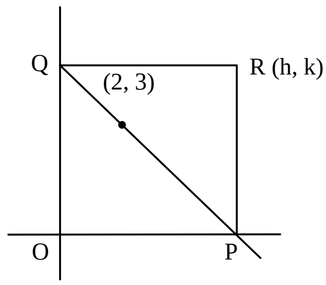
$$\Rightarrow P=\left(\frac{2m-3}{m},0\right),\ Q=(0,-(2m-3))$$

$$\therefore\ h=\frac{2m-3}{m},\ k=3-2m$$

$mh=2m-3,\ m=\dfrac{3-k}{2}$ ...(2)

$m(2-h)=3$

$m=\dfrac{3}{2-h}$ ...(1)



(1) = (2)

$$\Rightarrow \frac{3}{2-h}=\frac{3-k}{2}$$

$\Rightarrow (3-k)(2-h)=6$

$6-3h-2k+hk=6$

$\Rightarrow 3h+2k=hk$

$\therefore$ Locus of R is $3x+2y=xy$

26. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$

If B – 2A = 100 λ, then λ is equal to:

(1) 496 (2) 232 (3) 248 (4) 464

Ans (3)

Let $A = S_{20} = (1^2 + 3^2 + ... + 19^2) + (2.2^2 + 2.4^2 + ... + 2.20^2)$

$= (1^2 + 2^2 + 3^2 + ... + 20^2) - (2^2 + 4^2 + ... + 20^2) + 2(2^2 + 4^2 + ... + 20^2)$

$$A = \frac{20 \times 21 \times 41}{6} + \frac{2^2 \times 10 \times 11 \times 21}{6}$$

$B = S_{40} = (1^2 + 3^2 + ... + 39^2) + 2(2^2 + 4^2 + ... + 40^2)$

$$= \frac{40 \times 41 \times 81}{6} + \frac{2^2 \times 20 \times 21 \times 41}{6}$$

Now $B - 2A = 24800 = 100\lambda$

$\therefore \ \lambda = 248$

27. Let $y = y(x)$ be the solution of the differential equation $\sin x \frac{dy}{dx} + y \cos x = 4x, \ x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $-\frac{4}{9}\pi^2$ (2) $\frac{4}{9\sqrt{3}}\pi^2$ (3) $\frac{-8}{9\sqrt{3}}\pi^2$ (4) $-\frac{8}{9}\pi^2$

Ans (4)

$$\sin x \frac{dy}{dx} + y \cos x = 4x$$

$$\Rightarrow \frac{dy}{dx} + y \cot x = \frac{4x}{\sin x}$$

In the form of $\frac{dy}{dx} + Py = Q$

So I.F. = $e^{\int \cot x \, dx} = e^{\log \sin x} = \sin x$

Hence solution of the differential equation

y(I.F) = $\int Q(\text{I.F}) dx$

$\Rightarrow$ y sin x = $\int \frac{4x}{\sin x} \times \sin x \, dx$

$\Rightarrow y \sin x = 4 \cdot \frac{x^2}{2} + c$

$\Rightarrow$ y sin x = $2x^2$ + c … (1)

Given $y\left(\frac{\pi}{2}\right) = 0 \Rightarrow x = \frac{\pi}{2}$ and $y = 0$

$\Rightarrow 0 \times 1 = 2 \times \left(\frac{\pi}{2}\right)^2 + c$

$0 = \frac{\pi^2}{2} + c$

$c = -\frac{\pi^2}{2}$

At $x = \frac{\pi}{6}$, $y = ?$

Putting value of x in equation (1)

$$y.\sin\frac{\pi}{6} = 2\cdot\left(\frac{\pi}{6}\right)^2 - \frac{\pi^2}{2}$$

$$\Rightarrow\ y\times\frac{1}{2} = 2\times\frac{\pi^2}{36} - \frac{\pi^2}{2}$$

$$= \frac{\pi^2}{18} - \frac{\pi^2}{2}$$

$$= \frac{-8\pi^2}{18}$$

$$y = \frac{-8\pi^2}{9}$$

28. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, x + y + z = 7 is

(1) $\sqrt{\frac{2}{3}}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{2}{3}$ (4) $\frac{1}{3}$

Ans (1)

Equation of the line AC is

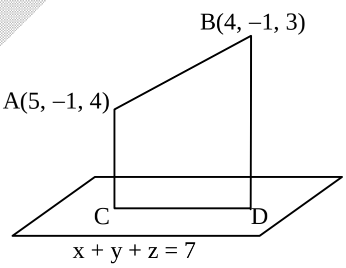
$$\frac{x-5}{1} = \frac{y+1}{1} = \frac{z-4}{1} = t$$

$\therefore$ Point C can be (t + 5, t – 1, t + 4)

This lies on the plane x + y + z = 7

$\therefore$ 3t + 8 = 7 $\therefore$ $t = -\frac{1}{3}$

$\therefore$ C is $\left(\frac{14}{3}, -\frac{4}{3}, \frac{11}{3}\right)$



Equation of the line BD is

$$\frac{x-4}{1} = \frac{y+1}{1} = \frac{z-3}{1} = t'$$

$\therefore$ Point D can be (4 + t′, –1 + t′, 3 + t′)

This lies on the plane x + y + z = 7

$\therefore$ 3t′ + 6 = 7 $\therefore$ $t' = \frac{1}{3}$

$\therefore$ D is $\left(\frac{13}{3}, -\frac{2}{3}, \frac{10}{3}\right)$

Length of projection = CD = $\sqrt{\frac{1}{9}+\frac{4}{9}+\frac{1}{9}} = \sqrt{\frac{2}{3}}$

29. Let $S = \{x \in R : x \geq 0$ and $2|\sqrt{x}-3| + \sqrt{x}(\sqrt{x}-6) + 6 = 0\}$. Then S

(1) contains exactly four elements.
(2) is an empty set.
(3) contains exactly one element.
(4) contains exactly two elements

Ans (4)

$$2|\sqrt{x}-3| + \sqrt{x}(\sqrt{x}-6) + 6 = 0$$

Case I: $0 \le x < 9$ i.e. $0 \le \sqrt{x} < 3$

$\therefore\ 2\left(-\sqrt{x}+3\right)+\sqrt{x}\left(\sqrt{x}-6\right)+6=0$

$-2\sqrt{x}+6+x-6\sqrt{x}+6=0$

$x-8\sqrt{x}+12=0$

i.e., $8\sqrt{x}=x+12$

$64x = x^2 + 14x + 144$

$64x^2 - 40x + 144 = 0$

$\Rightarrow x = 4$ or $x = 36$

But since $0 \le x < 9$

$\therefore\ x = 4$

Case II: $x \ge 9$ i.e. $\sqrt{x} \ge 3$

$2\left(\sqrt{x}-3\right)+\sqrt{x}\left(\sqrt{x}-6\right)+6=0$

$2\sqrt{x}-6+x-6\sqrt{x}+6=0$

$x-4\sqrt{x}=0$

$\Rightarrow x = 0$ or $x = 16$

But $x \ge 9$

$\therefore\ x = 16$

$\therefore\ S = \{4, 16\}$

$\therefore$ S has exactly two elements

30. Let $a_1, a_2, a_3, ..., a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + ... + a_{17}^2 = 140m$, then m is equal to

(1) 33 (2) 66 (3) 68 (4) 34

Ans (4)

$a_9 + a_{43} = 66.$

$\Rightarrow 2(a_1 + 25d) = 66$

$\Rightarrow a_1 + 25\ d = 33 \Rightarrow a_{26} = 33$ … (1)

$\sum_{k=0}^{12} a_{4k+1} = 416 \Rightarrow a_1 + a_5 + ... + a_{49} = 416$

$\Rightarrow a + 24\ d = 416$

$\Rightarrow a_{25} = 416$ … (2)

From (1) and (2)

$a = 8,\ d = 1$

$\therefore\ a_1 = 8, a_2 = 9, a_3 = 10, \ldots, a_{17} = 24$

$a_1^2 + a_2^2 + ... + a_{17}^2 = 8^2 + 9^2 + 10^2 + ... + 24^2$

$= 1^2 + 2^2 + \ldots + 24^2 - (1^2 + 2^2 + \ldots + 7^2)$

$= \dfrac{24(24+1)(48+1)}{6} - \dfrac{7(8)(15)}{6}$

$= 4900 - 140$

$= 4760$

$= 140\ (34)$

$\therefore\ m = 34$

PHYSICS

31. Three concentric metal shells A, B and C of respective radii a, b and c (a < b < c) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is:

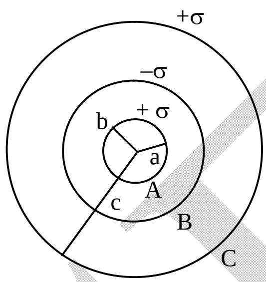
(1) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$ (2) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$ (3) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$ (4) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$

Ans (3)

$Q_A = \sigma(4\pi a^2)$

$Q_B = -\sigma(4\pi b^2)$

$Q_C = \sigma(4\pi c^2)$

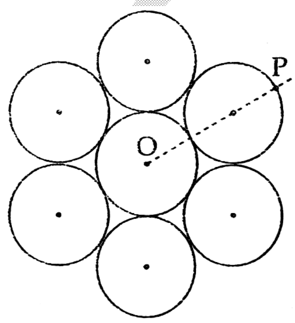


$$V_B = \frac{KQ_A}{b} + \frac{KQ_B}{b} + \frac{KQ_C}{c}$$

$$= \frac{K\sigma 4\pi a^2}{b} - \frac{K\sigma 4\pi b^2}{b} + \frac{K\sigma 4\pi c^2}{c}$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - \frac{b^2}{b} + c\right]$$

32. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is



(1) $\frac{181}{2}MR^2$

(2) $\frac{19}{2}MR^2$

(3) $\frac{55}{2}MR^2$

(4) $\frac{73}{2}MR^2$

Ans (1)

From theorem of parallel axis we can find moment of inertia of each of the 6 disc about O and that will be

$$I_1 = \frac{1}{2}MR^2 + M(2R)^2 = \frac{9}{2}MR^2$$

Hence, net moment of inertia about O

$$I_0 = 6I_1 + \frac{1}{2}MR^2$$

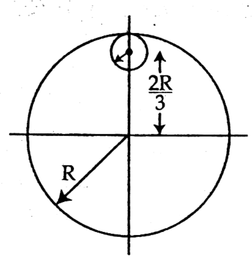
$$= \frac{54}{2}MR^2 + \frac{MR^2}{2} = \frac{55}{2}MR^2$$

$I_P = I_0 + 7M(3R)^2$

$$= \frac{55}{2}MR^2 + 63MR^2$$

$$= \frac{181}{2}MR^2$$

33. From a uniform circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is:



(1) $\frac{37}{9}MR^2$

(2) 4 MR^2

(3) $\frac{40}{9}MR^2$

(4) 10 MR^2

Ans (2)

$$I = \frac{1}{2}(9M)R^2 - \left[\frac{1}{2}M\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^2\right]$$

$$= \frac{9}{2}MR^2 - \frac{MR^2}{18} - \frac{4MR^2}{9}$$

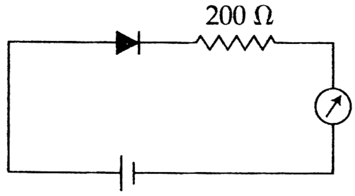
$$= \frac{81}{18}MR^2 - \frac{MR^2}{18} - \frac{8}{18}MR^2$$

$$= \frac{81}{18}MR^2 - \frac{9}{18}MR^2$$

$$= \frac{72}{18}MR^2$$

$$= 4\ MR^2$$

34. The reading of the ammeter for a silicon diode in the given circuit is



(1) 13.5 mA (2) 0 (3) 15 mA (4) 11.5 mA

Ans (4)

3V – 0.7 V = I (200 Ω)

$$\therefore \quad I = \frac{2.3\ V}{200\ \Omega} = 11.5\ mA$$

35. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is:

(1) 60° (2) 0° (3) 30° (4) 45°

Ans (4)

Concept: Malu's law

$I = I_m \cos^2(\theta)$

$\Rightarrow$ Angle between A and C is 45°

$\Rightarrow \quad I_C = \left(\frac{I}{2}\right)\cos^2(45°)$

$= \frac{I}{4}$

Angle between axes of B and C is also 45°

$\Rightarrow \quad I_B = I_C \cos^2(45°)$

$= \frac{I}{4} \times \frac{1}{2} = \frac{I}{8}$

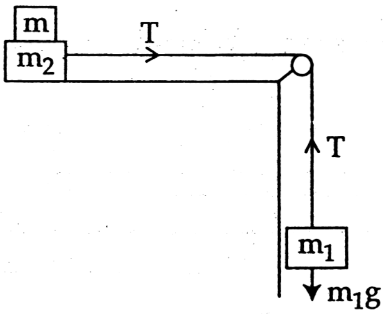
36. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by:

(1) $\frac{CR}{\omega_0}$ (2) $\frac{\omega_0 L}{R}$ (3) $\frac{\omega_0 R}{L}$ (4) $\frac{R}{(\omega_0 C)}$

Ans (2)

Conceptual.

37. Two masses m_1 = 5 kg and m_2 = 10 kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is:



(1) 10.3 kg (2) 18.3 kg (3) 27.3 kg (4) 43.3 kg

Ans (3)

Assuming there is sufficient friction between m and m_2, $\mu(m + m_2)g = m_1g$

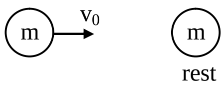
$\Rightarrow \quad m = \frac{m_1}{\mu} - m_2 = \frac{5}{0.15} - 10 = 23.33$ kg

Out of the four options, the minimum mass is 27.3 kg

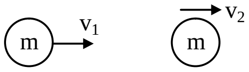
38. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is:

(1) $\frac{v_0}{\sqrt{2}}$ (2) $\frac{v_0}{4}$ (3) $\sqrt{2}\,v_0$ (4) $\frac{v_0}{2}$

Ans (3)



Before collision



After collision

Since kinetic energy increases, it is a super-elastic collision.

$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = 1.5 \times \frac{1}{2}mv_0^2$$

$$\Rightarrow \quad v_1^2 + v_2^2 = 1.5v_0^2$$

Momentum conservation gives

$$v_1 + v_2 = v_0$$

$$v_1^2 + v_2^2 + 2v_1v_2 = v_0^2$$

$$1.5v_0^2 + 2v_1v_2 = v_0^2$$

$$\Rightarrow \quad v_1v_2 = -\frac{v_0^2}{4}$$

$$(v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2$$

$$= v_0^2 - 4 \times \left(-\frac{v_0^2}{4}\right) = 2v_0^2 \Rightarrow v_1 - v_2 = \sqrt{2}v_0$$

39. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n[th] power of R. if the period of rotation of the particle is T, then:

(1) $T \propto R^{n/2}$ (2) $T \propto R^{3/2}$ for any n (3) $T \propto R^{\frac{n}{2}+1}$ (4) $T \propto R^{\frac{(n+1)}{2}}$

Ans (4)

$$\frac{k}{R^n} = m\omega^2 R = m \times \frac{4\pi^2}{T^2} \times R \Rightarrow T \propto R^{\frac{n+1}{2}}$$

40. Two batteries with emf 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between:

(1) 11.7 V and 11.8 V (2) 11.6 V and 11.7 V

(3) 11.5 V and 11.6 V (4) 11.4 V and 11.5 V

Ans (3)

$$\frac{E_1}{r_1} + \frac{E_2}{r_2} = \frac{E_0}{r_0} \quad \text{and} \quad \frac{1}{r_0} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$\frac{1}{r_0} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}$$

$$\frac{12}{1} + \frac{13}{2} = E_0 \times \frac{3}{2} \Rightarrow E_0 = \frac{37}{3}V$$

$$I = \frac{E_0}{R + r_0} = \frac{37/3\ V}{10\ \Omega + \frac{2}{3}\Omega} = \frac{37}{32}A$$

V across 10 Ω = $\frac{37}{32}A \times 10\Omega = 11.56$ V

41. In an ac circuit, the instantaneous e.m.f. and current are given by

$e = 100 \sin 30t$

$i = 20\sin\left(30t - \frac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively

(1) 50, 0 (2) 50, 10 (3) $\frac{1000}{\sqrt{2}}$, 10 (4) $\frac{50}{\sqrt{2}}$, 0

Ans (3)

Avg power delivered by an AC circuit

$$P_{avg} = V_0 I_0 \cos\phi = \frac{20\times100}{2}\times\frac{1}{\sqrt{2}} = \frac{1000}{\sqrt{2}} W$$

$$\therefore \quad i_{wattless} = i_{rms} \sin\frac{\pi}{4} = 20\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}} = 10 \text{ A}$$

42. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\,\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c} - t\right)\right]$ $\vec{E}_2 = E_{02}\,\hat{x}\cos[k(2z - ct)]$ in medium, where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic. If $\in_{r_1}$ and $\in_{r_2}$ refer to relative permittivities of air and medium respectively, which of the following options is correct?

(1) $\frac{\in_{r_1}}{\in_{r_2}} = \frac{1}{2}$ (2) $\frac{\in_{r_1}}{\in_{r_2}} = 4$ (3) $\frac{\in_{r_1}}{\in_{r_2}} = 2$ (4) $\frac{\in_{r_1}}{\in_{r_2}} = \frac{1}{4}$

Ans (4)

Speed of first wave, $v_0 = \frac{\omega_0}{k_0} = \frac{2\pi\nu}{\frac{2\pi\nu}{c}} = c$ … (1)

Speed of second wave, $v_1 = \frac{\omega_1}{k_1} = \frac{kc}{2k} = \frac{c}{2}$ … (2)

$$v_1 = \frac{1}{\sqrt{\mu_0 \in_0 \cdot \mu_r \in r_1}} \quad \ldots (3)$$

$$v_2 = \frac{1}{\sqrt{\mu_0 \in_0 \cdot \mu_r \in r_2}} \quad \ldots (4)$$

$$\frac{v_2}{v_1} = \sqrt{\frac{\in_{r_1}}{\in_{r_2}}} = \frac{\frac{c}{2}}{c} = \frac{1}{2}$$

$$\boxed{\frac{\in_{r_1}}{\in_{r_2}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}}$$

43. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?

(1) 2×10^6 (2) 2×10^3 (3) 2×10^4 (4) 2×10^5

Ans (4)

Available carrier frequency = 10 GHz

Utilized part $= \frac{10}{100} \times 10 \times 10^9 = 10^9$ Hz

Number of channels each requiring a bandwidth of 5 kHz is $= \frac{10^9 \text{ Hz}}{5 \times 10^3 \text{ Hz}} = 2 \times 10^5$

44. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg m^{-3} and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations?

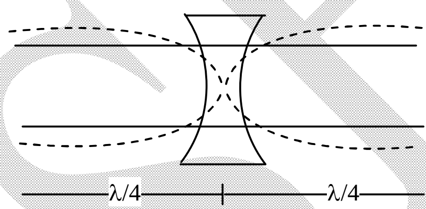
(1) 7.5 kHz (2) 5 kHz (3) 2.5 kHz (4) 10 kHz

Ans (2)

For fundamental mode

$\frac{\lambda}{2} = 60$ cm

$\lambda = 120$ cm



$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^3}}$

$= \sqrt{\frac{103}{3} \times 10^6}$

$n = \frac{v}{\lambda} = \sqrt{\frac{103}{3}} \times \frac{10^3}{120 \times 10^{-2}}$

$= \sqrt{\frac{103}{3} \times \frac{1}{12} \times 10^{14}}$

$= 0.488 \times 10^4$ Hz

≈ 5 kHz

45. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The values of p_d and p_c are respectively:

(1) (0, 1) (2) (0.89, 0.28) (3) (0.28, 0.89) (4) (0, 0)

Ans (2)

$m \times u = mv_1 + 2mv_2$

$\Rightarrow v_1 + 2v_2 = u$

$v_2 - v_1 = u \Rightarrow v_2 = \frac{2u}{3}$

Kinetic energy of deuterium $= \frac{1}{2} \times 2m\left(\frac{2u}{3}\right)^2 =$ Loss in energy of neutron

Fractional loss, $p_d = \frac{2 \times \frac{4}{9} \times \frac{1}{2}mu^2}{\frac{1}{2}mu^2} = 0.89$

$m \times u = mv_1 + 12mv_2$

$\Rightarrow v_1 + 12v_2 = u$

$v_2 - v_1 = u \Rightarrow v_2 = \frac{2u}{13}$

Kinetic energy of carbon atom $= \frac{1}{2} \times 12m \times \left(\frac{2u}{13}\right)^2$

Fractional loss $p_c = 12 \times \frac{4}{169} = 0.28$

46. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is

(1) 6% (2) 2.5% (3) 3.5% (4) 4.5%

Ans (4)

$$\rho = \frac{M}{V} = \frac{M}{l^3}$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + 3\frac{\Delta l}{l}$$

$$\frac{\Delta\rho}{\rho}(100) = 1.5 + 3(1) = 4.5\%$$

47. Two moles of an ideal monoatomic gas occupies a volume V at 27 °C. The gas expands adiabatically to a volume 2 V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

(1) (a) 195 K (b) 2.7 kJ

(2) (a) 189 K (b) 2.7 kJ

(3) (a) 195 K (b) –2.7 kJ

(4) (a) 189 K (b) –2.7 kJ

Ans (4)

$$TV^{\gamma-1} = \text{constant}$$

$$(300)V^{\frac{2}{3}} = T(2V)^{\frac{2}{3}}$$

$$T = \frac{300}{4^{\frac{1}{3}}} = \frac{300}{1.587} = 189\text{ K}$$

$$\Delta U = nC_V\Delta T = (2)\left(\frac{3}{2} \times 8.314\right)(111) = 25 \times 111 = 2.7\text{ kJ}$$

As temperature increases ΔU is negative

48. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$ is

(1) $\frac{mg}{Ka}$ (2) $\frac{Ka}{mg}$ (3) $\frac{Ka}{3\,mg}$ (4) $\frac{mg}{3\,Ka}$

Ans (4)

Pressure applied (P) $= \frac{mg}{a}$

Bulk modulus (K) $= \frac{P}{-\frac{dV}{V}}$

or $-\frac{dV}{V}=\frac{P}{K}=\frac{mg}{aK}$

Now $V=\frac{4}{3}\pi R^3$

$\therefore\ \frac{dV}{V}=\frac{3dr}{r}$

or $\frac{dr}{r}=\frac{1}{3}\frac{dV}{V}=\frac{1}{3}\frac{mg}{aK}$

49. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K=\frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be:

(1) 0.9 nC (2) 1.2 nC (3) 0.3 nC (4) 2.4 nC

Ans (2)

$q_{ind}=q\left[1-\frac{1}{K}\right]$

$q=KCV$

$=\frac{5}{3}\times 90\times 20\times 10^{-12}C$

$K=\frac{5}{3}$

$q_{in}=-1.2\times 10^{-9}C$

$|q_{ind}|=1.2\ nC$

50. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is:

(1) $\frac{1}{\sqrt{2}}$ (2) 2 (3) $\sqrt{3}$ (4) $\sqrt{2}$

Ans (4)

Given: $\mu=i\times A$

Keeping current constant μ is doubled

$\Rightarrow$ Area is doubled $\Rightarrow$ radius becomes $\sqrt{2}R_0$

Now $B\propto\frac{1}{R}$ for a ring

$\therefore$ If $R'=\sqrt{2}R_0$

$\therefore\ B'=\frac{B}{\sqrt{2}}\Rightarrow\frac{B}{B'}=\sqrt{2}$

51. An electron from various excited states of hydrogen atom emits radiation to come to the ground state. Let λ_n, λ_g be the de Brogue wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants)

(1) $\Lambda_n^2 \approx \lambda$ (2) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$ (3) $\Lambda_n \approx A + B\lambda_n$ (4) $\Lambda_n^2 \approx A + B\lambda_n^2$

Ans (2)

$$\frac{1}{\Lambda_n} = R\left(1-\frac{1}{n^2}\right) \Rightarrow \Lambda_n = \frac{1}{R}\left(1-\frac{1}{n^2}\right)^{-1}$$

$$\Lambda_n = \frac{1}{R} + \frac{1}{R_n^2}$$ (Use Binomial for large n)

But $\lambda_{deBroglie} \propto \frac{1}{\nu_n}$ and $\nu_n \propto \frac{1}{n}$

$\therefore\ \lambda_{deBroglie} \propto n$

$$\therefore \Lambda_n = \frac{1}{R} + \frac{B}{R_n^2}$$

or $\Lambda_n = A + \frac{B}{\lambda_n^2}$

52. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, arid rebound elastically with a speed of 10^3 ms^{-1}, then the pressure on the wall is nearly

(1) 4.70×10^2 Nm^{-2} (2) 2.35×10^3 Nm^{-2} (3) 4.70×10^3 Nm^{-2} (4) 2.35×10^2 Nm^{-2}

Ans (2)

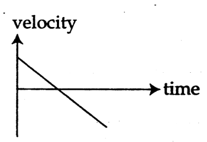
$$F = \frac{dP}{dt} = \frac{dn}{dt}\left[2mv\sin(45°)\right]$$

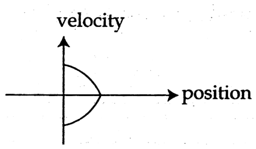
$$P = \frac{F}{A} = \frac{dn}{dt}\frac{\left[2mv\sin(45°)\right]}{A}$$

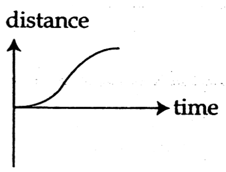
$$= \frac{10^{23} \times 3.32 \times 10^{-27} \times 2 \times 10^3}{2 \times 10^{-4} \times \sqrt{2}}$$

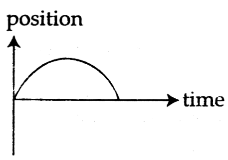
$= 2.35 \times 10^3$ Nm^{-2}

53. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up

(1) 

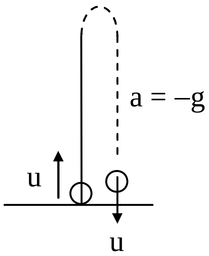
(2) 

(3) 

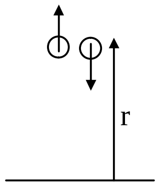
(4) 

Ans (3)

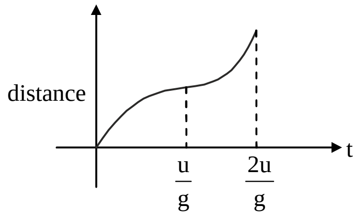
The motion is under constant acceleration. As an example take a following situation.



(1)



(2)



54. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_P, r_α respectively in a uniform magnetic field B. The relation between r_e, r_P, r_α is:

(1) $r_e < r_\alpha < r_P$ (2) $r_e > r_P = r_\alpha$ (3) $r_e < r_P = r_\alpha$ (4) $r_e < r_P < r_\alpha$

Ans (3)

$$\frac{mv^2}{r} = Bq^2 \Rightarrow r = \frac{mv}{Bg} = \frac{p}{Bq} = \frac{\sqrt{2mE}}{Bq} \propto \frac{\sqrt{m}}{q}$$

$$r_e \propto \frac{\sqrt{m_e}}{e}$$

$$r_p \propto \frac{\sqrt{m_p}}{e}$$

$$r_\alpha \propto \frac{\sqrt{4mp}}{2e} = \frac{\sqrt{m_p}}{e}$$

$\therefore\ r_e < r_P = r_\alpha$

55. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances?

(1) 910 Ω (2) 990 Ω (3) 505 Ω (4) 550 Ω

Ans (4)

$$\frac{R_1}{R_2} = \frac{l_1}{100 - l_1} \Rightarrow \frac{R_1 + R_2}{R_2} = \frac{100}{100 - l_1} \quad \ldots (1)$$

$$\frac{R_2}{R_1} = \frac{l_1 - 10}{110 - l_1} \Rightarrow \frac{R_1 + R_2}{R_2} = \frac{100}{l_1 - 10} \quad \ldots (2)$$

From (1) and (2)

$100 - l_1 = l - 10$

$\Rightarrow\ 2l_1 = 55\,\text{cm}$

$\Rightarrow\ \because\ R_1 + R_2 = 1\ k\,\Omega$

$$\Rightarrow \frac{1}{R_2} = \frac{100}{100 - 55} \Rightarrow R_2 = \frac{45}{100}\ k\Omega \text{ and } R_1 = \left(1 - \frac{45}{100}\right) k\Omega = 550\,\Omega$$

56. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

(1) 2.5 Ω (2) 1 Ω (3) 1.5 Ω (4) 2 Ω

Ans (3)

$$\frac{\varepsilon_1}{V_1} = \frac{52\text{ cm}}{40\text{ cm}}; \qquad V_1 = \varepsilon_1 - ir_1$$

$$= \varepsilon_1 - \frac{\varepsilon_1}{R + r_1} \times r_1$$

$$= \varepsilon_1 \left[\frac{R}{R + r_1}\right]$$

$$\frac{\varepsilon_1}{\varepsilon_1\left[\frac{R}{R + r_1}\right]} = \frac{52}{40} = 1.3$$

$$\frac{R + r_1}{R} = 1.3$$

$5 + r_1 = 1.3 \times 5$

$\Rightarrow r_1 = 1.5\ \Omega$

Alternate method

$$r = \left(\frac{l_1 - l_2}{l_2}\right)R$$

$$= \left(\frac{52 - 40}{40}\right) \times 5 = 1.5\ \Omega$$

57. If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is

(1) $\frac{\nu_L}{25}$ (2) $25\ \nu_L$ (3) $16\ \nu_L$ (4) $\frac{\nu_L}{16}$

Ans (1)

$$\nu_L = \frac{c}{\lambda_{Ly}} = R\left[\frac{1}{n_1^2} - \frac{1}{\infty}\right] \quad [\because n_1 = 1]$$

$$\nu_L = \frac{c}{\lambda_{Ly}} = R$$

Pfund Series

$$\nu_{Pfund} = \frac{c}{\lambda_P} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \quad \begin{matrix} n_2 = \infty \\ n_1 = 5 \end{matrix}$$

$$\nu_{Pfund} = \frac{R}{25} = \frac{\nu_L}{25}$$

58. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance? (i.e., distance between the centres of each slit.)

(1) 100 μm (2) 25 μm (3) 50 μm (4) 75 μm

Ans (2)

Position of first minima = $\frac{60^\circ}{2} = 30^\circ$

$a \sin 30^\circ = \lambda$

$a \to$ slit width = 1×10^{-6} m

$\Rightarrow \lambda = \frac{1}{2} \times 10^{-6}$ m

$\beta = \frac{D\lambda}{d}$

$\beta = 10^{-2}$ m, D = 0.5 m

$d = \frac{D\lambda}{\beta} = 25\ \mu m$

59. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is:

(1) $-\frac{3}{2}\frac{k}{a^2}$ (2) $-\frac{k}{4a^2}$ (3) $\frac{k}{2a^2}$ (4) zero

Ans (4)

$F = -\frac{dU}{dr} = -\frac{d}{dr}\left(-\frac{K}{2r^2}\right)$

$= +\frac{R}{2}\frac{(-2)}{r^3} = -\frac{K}{r^3}$

$\frac{mv^2}{r} = \frac{R}{r^3} \Rightarrow \frac{1}{2}mv^2 = \frac{K}{2r^2}$

Total energy K + U = $\frac{K}{2r^2} - \frac{K}{2r^2} = 0$

60. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of $10^{12}\ s^{-1}$. What is the force constant of the bonds connecting one atom with the other?

(Molecular weight of silver = 108 and Avagadro number = 6.02×10^{23} gm $mole^{-1}$)

(1) 5.5 Nm^{-1} (2) 6.4 Nm^{-1} (3) 7.1 Nm^{-1} (4) 2.2 Nm^{-1}

Ans (3)

We know for SHM,

Corresponding $\omega_n = \sqrt{\frac{k}{m}}$ or, $k = \omega_n^2 \times m$ $\left[\text{mass of 1 Ag atom in grams} = \frac{\text{Molecular weight}}{\text{Number of atoms in 1 g mole}}\right]$

or, $k = 4\pi^2 \times (10^{12})^2 \times \frac{108 \times 10^{-3}}{6.023 \times 10^{23}}$

$k = 7.1\ Nm^{-1}$

CHEMISTRY

61. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?

(1) $[Co(H_2O)_3C1_3]3H_2O$ (2) $[Co(H_2O)_6]Cl_3$

(3) $[Co(H_2O)_5C1]C1_2.\ H_2O$ (4) $[Co(H_2O)_4Cl_2]Cl.2H_2O$

Ans (1)

$\Delta T_f = k_f.m.i$

(1) i for i = 1

(2) i for $[Co(H_2O)]Cl_3 \rightarrow [Co(H_2O)_6]^{3+} + 3Cl^- \Rightarrow i = 4$

(3) i for i = 3

(4) i for i = 2

Lower the value of i higher will be the freezing point

1 molal $[Co(H_2O)_3Cl_3].3H_2O$ has highest freezing point

62. Hydrogen peroxide oxidises $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are, respectively

(1) H_2O and $(H_2O + OH^-)$ (2) $(H_2O + O_2)$ and H_2O

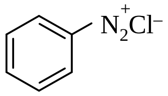
(3) $(H_2O + O_2)$ and $(H_2O + OH^-)$ (4) H_2O and $(H_2O + O_2)$

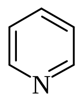
Ans (4)

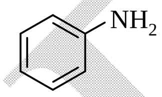
$$H_2\overset{-1}{O_2} + [\overset{+2}{Fe}(CN)_6]^{4-} \xrightarrow{H^+} [\overset{+3}{Fe}(CN)_6]^{3-} + H_2\overset{-2}{O}$$

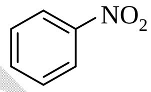
$$H_2\overset{-1}{O_2} + [\overset{+3}{Fe}(CN)_6]^{3-} \xrightarrow{OH^-} [\overset{+2}{Fe}(CN)_6]^{4-} + H_2O + \overset{0}{O_2}$$

63. Which of the following compounds will be suitable for Kjeidahl's method for nitrogen estimation?

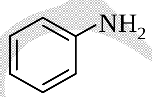
(1) 

(2) 

(3) 

(4) 

Ans (3)



Because Kjeldahl's method does not work for

(a) diazocompounds (b) nitrocompounds (c) nitrogen a part of ring.

64. Glucose on prolonged heating with HI gives

(1) 6-iodohexanal (2) *n*-Hexane (3) 1-Hexene (4) Hexanoic acid

Ans (2)

On prolonged heating with concentrated HI and red phosphorous at 383 K, glucose *n*-hexane

$$CH_2OH(CHOH)_4CHO \xrightarrow{\text{HI, heat}} \underset{n\text{-hexene}}{CH_3-(CH_2)_4-CH_3}$$

65. An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination?

	Base	Acid	Endpoint
(1)	Strong	Strong	Pink to Colourless
(2)	Weak	Strong	Colourless to Pink
(3)	Strong	Strong	Pinkish red to yellow
(4)	Weak	Strong	Yellow to pinkish red

Ans (4)

Methyl orange: Basic $\rightarrow$ yellow colour

Acidic $\rightarrow$ red colour

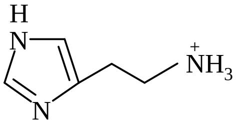
Range: 3.1 – 4.4

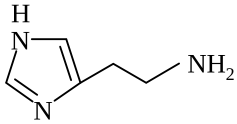
For methyl orange to be used; range of pH should fall between : 3.1 – 4.4 which is acidic

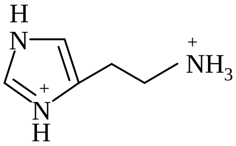
Hence, Base $\rightarrow$ weak

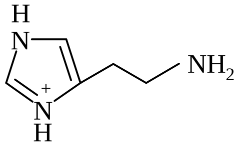
Acid $\rightarrow$ strong

66. The predominant form of histamine present in human blood is (PK_a, Histidine = 6.0)

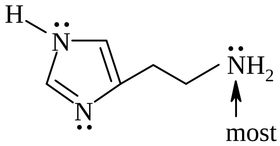
(1) 

(2) 

(3) 

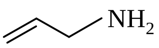
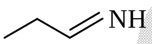
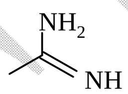
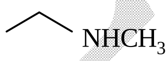
(4) 

Ans (1)



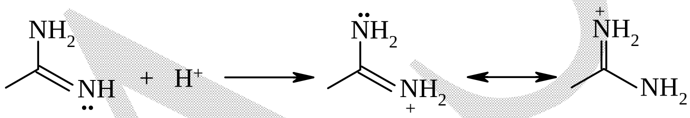
most basic, therefore gets
protonated perferentially

67. The increasing order of basicity of the following compounds is:

(a) 
(b) 
(c) 
(c) 

(1) (d) < (b) < (a) < (c)
(2) (a) < (b) < (c) <(d)
(3) (b) < (a) < (c) < (d)
(4) (b) < (a) < (d) < (c)

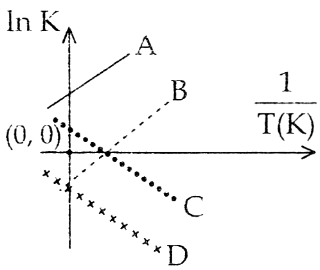
Ans (4)



The conjugate acid of base (c) is resonance stabilized, so (c) is relatively stronger acid.

The sp^2 hybridised nitrogen, due to higher “s” character, is relatively more electronegative. This makes the electron density of lone pair will be less, that makes (b) the weakest base.
Therefore correct increasing order of basicity is (b) < (a) < (d) < (c).

68. Which of the following lines correctly show the temperature dependence of equilibrium constant, K, for an exothermic reaction ?



(1) A and D
(2) A and B
(3) B and C
(4) C and D

Ans (2)

RT ln K = – ΔG°

$$\Rightarrow l\text{n}\,K = \frac{(-\Delta H^\circ + T\Delta S^\circ)}{RT}$$

$$l\text{n}\,K = \left(-\frac{\Delta H^\circ}{R}\right)\left(\frac{1}{T}\right) + \frac{\Delta S^\circ}{R}$$

$$\updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow$$

$$y = m \cdot x + c$$

Slope $= m = \frac{-\Delta H^\circ}{R}$ ⇒ for an exothermic reaction, $\Delta H < 0$,

Slope = – ve × – ve = positive

The graph of ln K is $\left(\frac{1}{T}\right)$ should be a straight line with a positive slope. Lines A and B are straight lines with positive slope, so correct options are A and B, option (2).

69. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane? (Atomic weight of B =10.8 u)

(1) 1.6 hours (2) 6.4 hours (3) 0.8 hours (4) 3.2 hours

Ans (4)

$$B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$$

2.76 g of diborane required 96 g of O_2 so mass of oxygen (w) = 96 g

According to Faraday's first law

w = ZIt

$$96 = \frac{8}{96500} \times 100 \times t$$

$$t = \frac{96 \times 96500}{8 \times 100}$$

$$= 11580 \text{ s}$$

$$= \frac{11580}{3600} \text{hr}$$

$$\boxed{t = 3.2\,\text{hr}}$$

70. Consider the following reaction and statements

$$[Co(NH_3)_4Br_2]^+ + Br^- \rightarrow [Co(NH_3)_3Br_3] + NH_3$$

(I) Two isomers are produced if the reactant complex ion is a *cis*-isomer.

(II) Two isomers are produced if the reactant complex ion is a *trans*isomer.

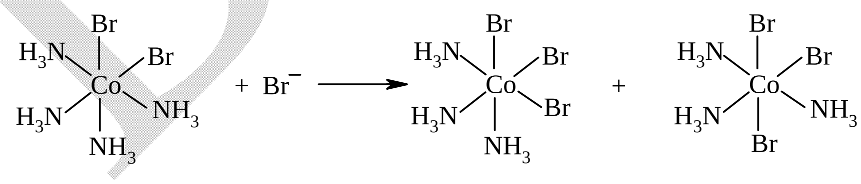
(Ill) Only one isomer is produced if the reactant complex ion is a *trans*isomer.

(IV) Only one isomer is produced if the reactant complex ion is a *cis*-isomer.

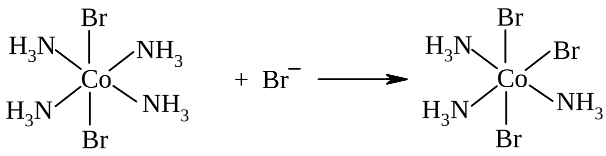
The correct statements are

(1) (II) and (IV) (2) (1) and (II) (3) (I) and (III) (4) (III) and (IV)

Ans (3)

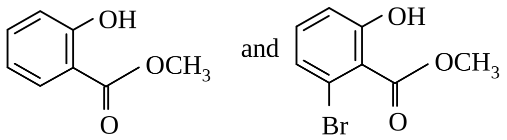


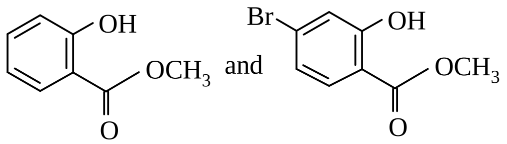
cis-form facial meridional

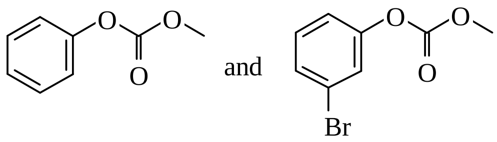


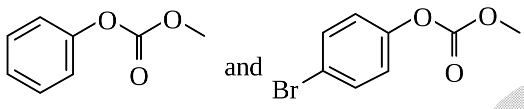
trans-form one possible isomer

71. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with Br_2 to form product B. A and B are respectively:

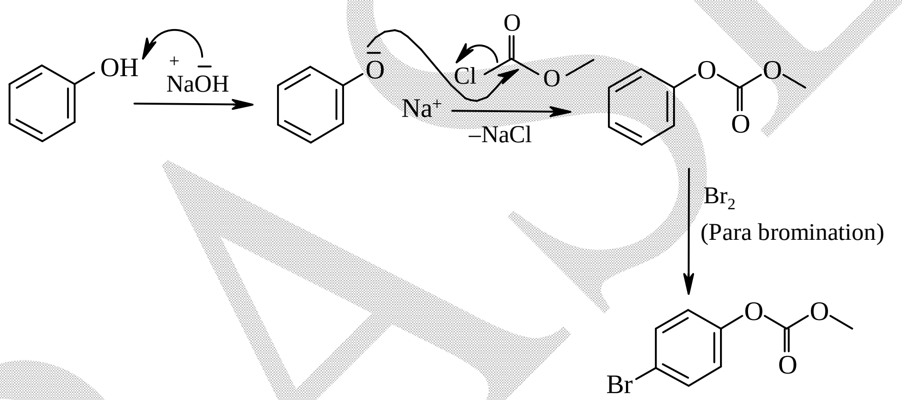
(1) 

(2) 

(3) 

(4) 

Ans (4)



72. An aqueous solution contains an unknown concentration of Ba^{2+}. When 50 mL of a I M solution of Na_2SO_4 is added, $BaSO_4$ just begins to precipitate. The final volume is 500 mL. The solubility product of BaSO4 is 1×10^{-10}. What is the original concentration of Ba^{2+} ?

(1) 1.0×10^{-10} M (2) 5×10^{-9} M (3) 2×10^{-9} M (4) 1.1×10^{-9} M

Ans (4)

Given, Concentration of Na_2SO_4 = 1 M of 50 ml = 50 × 1 = 50 mmoles

Concentration of Na_2SO_4 in 500 ml solution $= \frac{50}{500} = 10^{-1}$ M

$$BaSO_4 \rightleftharpoons Ba^{2+} + SO_4^{2-}$$

Let x be concentration of Ba^{2+}

$\therefore$ Concentration of Ba^{2+} in 500 ml solution is $\frac{x \times 450}{500}$

$\therefore K_{sp} = [Ba^{2+}]\left[SO_4^{2-}\right]$

$$\Rightarrow 10^{-10} = 10^{-1} \times \left[\frac{x \times 450}{500}\right] \Rightarrow \boxed{x = 1.1 \times 10^{-9}\ M}$$

73. At 518 °C, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was 1.00 Torr s^{-1} when 5% had reacted and 0.5 Torr s when 33% had reacted. The order of the reaction is

(1) 0 (2) 2 (3) 3 (4) 1

Ans (2)

Rate $\propto [A]^n$

$Rate_1 \propto (95)^n$

$Rate_2 \propto (67)^n$

$$\frac{1}{0.5} = \left(\frac{95}{67}\right)^n$$

$$2 = (1.4179)^n$$

log2 = nlog(1.4179)

$$n = \frac{\log 2}{\log(1.4179)}$$

$$= \frac{0.3010}{0.1516} = 1.985 \simeq 2$$

74. The combustion of benzene (l) gives $CO_2(g)$ and $H_2O(l)$. Given that heat of combustion of benzene at constant volume is –3263.9 kJ $mo1^{-1}$ at 25 °C; heat of combustion (in kJ mol^{-1}) of benzene at constant pressure will be: (R = 8.314 JK^{-1} mol^{-1})

(1) –3267.6 (2) 4152.6 (3) –452.46 (4) 3260

Ans (1)

Given, $\Delta U = -3263.9$ kJ mol^{-1}

$$C_6H_6(l) + \frac{15}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H_2O(l)$$

$$\Delta n_g = -\frac{3}{2} = -1.5$$

T = 25 °C = 298 K

$\Delta H = \Delta U + \Delta n_g RT$

$= 3263.9 + (-1.5) \times 8.31 \times 10^{-3} \times 298$

$= -3267.6$ kJ mol^{-1}

75. The ratio of mass percent of C and H of an organic compound ($C_XH_YO_Z$) is 6: 1. If one molecule of the above compound ($C_XH_YO_Z$) contains half as much oxygen as required to burn one molecule of compound C_XH_Y completely to CO_2 and H_2O. The empirical formula of compound $C_XH_YO_Z$ is:

(1) $C_2H_4O_3$ (2) $C_3H_6O_3$ (3) C_2H_4O (4) $C_3H_4O_2$

Ans (1)

	C	:	H
Mass %	6	:	1
Mole ratio	$\frac{1}{2}$	:	1
Mole ratio	1	:	2

$$C_XH_Y + \left(\frac{Y+4x}{2}\right)O_2 \longrightarrow XCO_2 + \frac{Y}{2}H_2O$$

$$Z = \frac{Y+4x}{2}$$

$$Z = \frac{Y + 4x}{4}$$

$$Z = \frac{2 + 4(1)}{4} = \frac{3}{2}$$

$C_1\ H_2\ O_{3/2}$

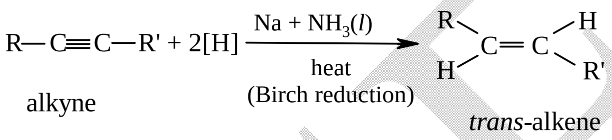
$C_2\ H_4\ O_3$

76. The trans-alkenes are formed by the reduction of alkynes with:

(1) Sn - HC1 (2) H_2 - Pd/C, $BaSO_4$ (3) $NaBH_4$ (4) Na/liq. NH_3

Ans (4)

Alkynes can be reduced to *trans*-alkenes using sodium in liquid ammonia (Na/liq.NH_3). This reaction is stereospecific giving only the *trans*-alkene via on *anti* addition



77. Which of the following are Lewis acids?

(1) BCl_3 and $AlCl_3$ (2) PH_3 and BCl_3 (3) $AlCl_3$ and $SiCl_4$ (4) PH_3 and $SiCl_4$

Ans (1)

Both BCl_3 and $AlCl_3$ have incomplete octet around the central atom i.e., B and Al respectively. Hence, they act as Lewis acids.

78. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal M' is:

(1) Fe (2) Zn (3) Ca (4) Al

Ans (4)

$$\underset{(M)}{Al} + 3NaOH \longrightarrow \underset{\substack{\text{White ppt.}\\(X)}}{Al(OH)_3} \downarrow + 3Na^+$$

$$\underset{(X)}{Al(OH)_3} \xrightarrow[\text{(excess)}]{\text{NaOH}} \underset{\text{sodium tetrahydroxoaluminate (III)}}{2\overset{+}{Na}[Al(OH)_4]^-}$$

$$\underset{(X)}{Al(OH)_3} \xrightarrow{\text{heat}} Al_2O_3 + 3H_2O$$

Alumina is used in chromatography as an adsorbent.

79. According to molecular orbital theory, which of the following will not be a viable molecule?

(1) H_2^{2-} (2) He_2^{2+} (3) He_2^{+} (4) H_2^{-}

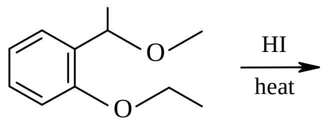
Ans (1)

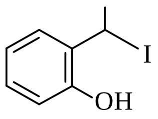
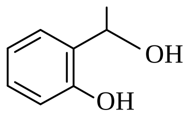
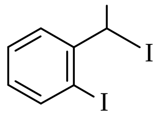
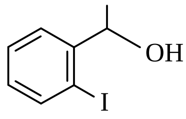
H_2^{2-}

Electronic configuration: $(\sigma_{1s})^2\ (\sigma^*_{1s})^2$

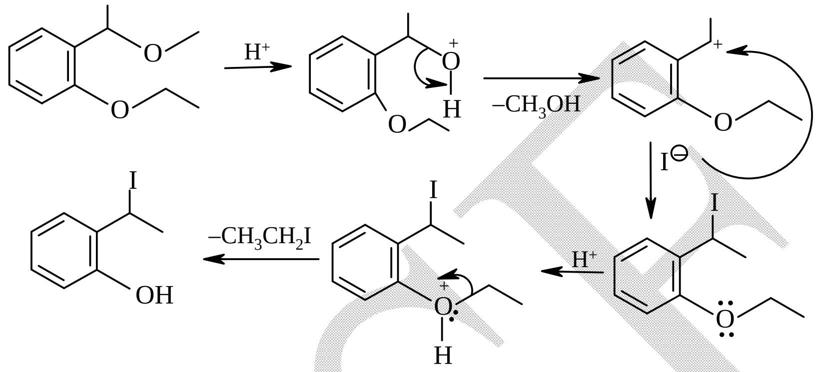
$\therefore$ Bond order = 0

80. The major product formed in the following reaction is:

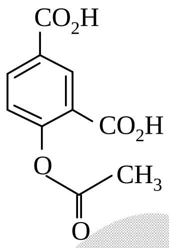


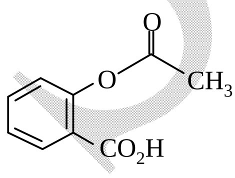
(1)  (2)  (3)  (4) 

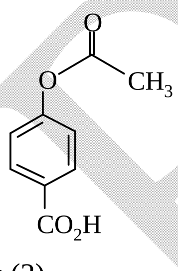
Ans (1)

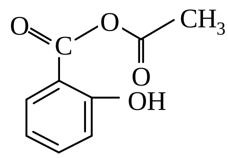


81. Phenol on treatment with CO_2 in the presence of NaOH followed by acidification produces compound X as the major product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of H_2SO_4 produces:

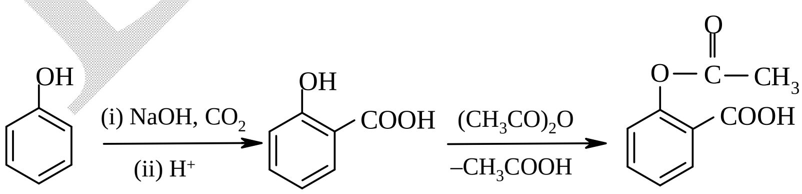
(1) 

(2) 

(3) 

(4) 

Ans (2)



(Kolbe's reaction)

82. Which of the following compounds contain(s) no covalent bond(s) ?

$KCl, PH_3, O_2, B_2H_6, H_2SO_4$

(1) KCl, B_2H_6 (2) KCl, B_2H_6, PH_3 (3) KCl, H_2SO_4 (4) KCl

Ans (4)

KCl is an ionic compound

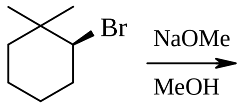
83. Which type of 'defect' has the presence of cations in the interstitial sites?

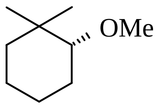
(1) Metal deficiency defect
(2) Schottky defect
(3) Vacancy defect
(4) Frenkel defect

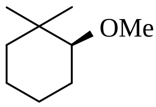
Ans (4)

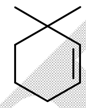
A Frenkel defect forms when an atom or smaller ion (usually cation) leaves its place in the lattice occupies interstitial sites.

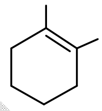
84. The major product of the following reaction is:



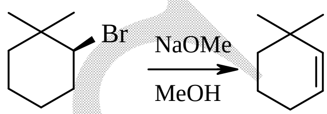
(1) 

(2) 

(3) 

(4) 

Ans (4)



Strong base, $NaOCH_3$ and the bulky substrate favours E_2. The Br and a *trans* proton to the leaving group are eliminated to get a pi bond.

85. The compound that does not produce nitrogen gas by the thermal decomposition is

(1) $(NH_4)_2SO_4$
(2) $Ba(N_3)_2$
(3) $(NH_4)_2Cr_2O_7$
(4) NH_4NO_2

Ans (1)

$$Ba(N_3)_2 \longrightarrow Ba + N_2$$

$$(NH_4)_2Cr_2O_7 \longrightarrow N_2 + Cr_2O_3 + H_2O$$

$$NH_4NO_2 \longrightarrow N_2 + H_2O$$

86. An aqueous solution contains 0.10 M H_2S and 0.20 M HCl. If the equilibrium constants for the formation of HS^- from H_2S is 1.0×10^{-7} and that of S^{2-} from HS ions is 1.2×10^{-13} then the concentration of S^{2-} ions in aqueous solution is

(1) 5×10^{-19}
(2) 5×10^{-8}
(3) 3×10^{-20}
(4) 6×10^{-21}

Ans (3)

$$H_2S \rightleftharpoons HS^- + H^+ \qquad K_{a_1} = 1\times10^{-7}$$

$$HS^- \rightleftharpoons S^{2-} + H^+ \qquad K_{a_2} = 1.2\times10^{-15}$$

$$K_{a_1} \times K_{a_2} = \frac{[H^+]^2[S^{2-}]}{[H_2S]}$$

$$10^{-7} \times 1.2\times10^{-13} = \frac{(0.2)^2(S^{2-})}{(0.1)}$$

$$\frac{10^{-18} \times 1.2\times10^{-13}}{0.2\times0.2} = [S^{2-}]$$

$[S^{2-}] = 30 \times 10^{-21}$

$$\boxed{= 3\times10^{-20}}$$

87. The oxidation states of Cr in $[Cr(H_2O)_6]Cl_3$, $[Cr(C_6H_6)_2]$, and $K_2[Cr(CN_2(O)_2(NH_3)$ respectively are

(1) 0, and +4 (2) +3, +4, and +6 (3) +3, +2, and +4 (4) +3, 0, and +6

Ans (4)

Oxidation state of Cr

$[Cr(H_2O)_6]Cl_3$

$x + 6(0) + 3(-1) = 0$

$\therefore x = 3$

$[Cr(C_6H_6)_2]$

$x + 2(0) = 0$

$\therefore x = 0$

$K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$

(i) Considering O_2 as oxygen molecule (neutral ligand)

$2(+1) + x + 2(-1) + 2(-2) + 0 + 0$

$x = +4$

Ans (1)

(ii) Considering (O_2) as peroxo lignad

$2(+1) + x + 2(-1) + 2(-2) + (-2) + 0 + 0$

$x = +6$

Ans (4)

88. The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting $[3Ca_3(PO_4)_2Ca(OH)_2]$ to

(1) $[3\{Ca(OH)_2\}CaF_2]$ (2) $[CaF_2]$ (3) $[3(CaF_2)Ca(OH)_2]$ (4) $[3Ca_3(PO_4)_2CaF_2]$

Ans (4)

The F^- ions make the enamel on teeth much harder by converting hydroxy apatite, $[3(Ca_3(PO_4)_2\cdot Ca(OH)_2]$ the enamel on the surface of the teeth, into much harder fluropatite, $[3(Ca_3(PO_4)_2 \cdot CaF_2]$.

89. Which of the following salts is the most basic in aqueous solution?

(1) $Pb(CH_3COO)_2$ (2) $A1(CN)_3$ (3) CH_3COOK (4) $FeCl_3$

Ans (3)

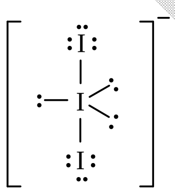
$$CH_3COOK + H_2O \longrightarrow CH_3COOH + KOH$$

CH_3COOK is a salt of strong base and weak acid.

90. Total number of lone pair of electrons I_3^- ion is

(1) 12 (2) 3 (3) 6 (4) 9

Ans (4)



I_3^- is a linear molecule. It contains 9 lone pairs of electrons.

* * *